Market for Multi-Dimensional Flexibility with Parametric Demand Response Bidding

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***Abstract*—Demand side management (DSM) exploits flexibility of the end-user side to help improve the performance of the power grid. However, how to model and evaluate the multi-dimensional flexibility (MDF) of the energy consumers (such as the flexibilities in terms of energy, power, time period and locations, etc.) is an important and challenging issue. To handle this, we propose a day-ahead market design for the MDF services. In this market, the MDF aggregators need to submit the characteristics of their virtual battery models and parametric reward functions, which represent the aggregate load flexibilities and the required remunerations, respectively. One key feature of this new market is that the system operator is allowed to accept only a portion of every bid instead of the whole of them. This can bring more flexibilities for the operator to minimize the total system cost. With various case studies on a 6-bus transmission network, we show that this market can help flatten the locational marginal prices (LMPs) across the peak load period. Moreover, we show that even if the MDF bids at different buses are the same, the clearing results can be quite different, which reflect the practical values of different load flexibilities from the perspective of the system operator.**

I. Introduction

Demand side management (DSM) is an important feature of smart grids, which exploits the flexibility of the energy consumers to help improve the performance of the electric grid [1]. In traditional power systems, it is the generation side that adapts the generator outputs to the change of the loads in order to maintain the balance between the supply and the demand. However, as more and more renewable energy sources (RES) are integrated into the grid, it becomes challenging to maintain the balance by solely relying on the supply side because of the intermittent nature of RES. In this regard, to efficiently leverage the flexibility of the electricity users is one of the promising schemes to help overcome the limits of traditional power systems.

DSM includes everything that is done on the demand side of an energy system, ranging from exchanging old incandescent light bulbs to compact fluorescent lights (CFLs) up to installing a sophisticated dynamic load management system [2]. Among those different kinds of DSM schemes, demand response (DR) is a main category which has been drawing a lot of attention [3], [4].

DR can be defined as the changes in electricity usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time [3]. There are mainly two categories of DR programs [4]. One is the price-based program (PBP) which uses dynamic pricing rates in order to induce the energy users to shift or curtail their loads such that the demand curve can be flattened. The other one is the incentive-based program (IBP) where direct load control is enabled such that the system operator has the authority to optimally manage the flexible loads. In return, the participants will be rewarded depending on the service they provide. Compared with PBP, IBP has some advantages. One of them is that those controllable loads are managed in a centralized way and hence, the aggregate benefits from all participants can be maximized in a systematic manner. While, in PBP, the system has to rely on the voluntary responses of the consumers to the price signals where lots of efforts need to be made for predicting the individual behavior of different energy users [5].

In both programs, the pricing problem is an important issue. One effective method is the smart pricing [6]–[8]. In [6], a real-time pricing algorithm for DSM programs is introduced to encourage desired energy consumption behaviors among users and to keep the total consumption level below the power generation capacity. In [7], a smart pricing method is proposed by using Vickrey-Clarke-Groves (VCG) mechanism. It aims to maximize the social welfare, i.e., the aggregate utility functions of all users minus the total energy cost, while guaranteeing the efficiency and user truthfulness at the equilibrium. Another type of method is market-based. In [8], a demand-side bidding mechanism is proposed, in which demand-side bids include Price Elasticity Matrices (PEM) that represent any hour's load responsiveness to prices across time periods within the market's time-frame. An algorithm is designed to help the market administrator determine the dispatch of the flexible loads and the market clearing price.

One limitation in the aforementioned work is that the network constraints have been ignored. In contrast, the transmission network constraints are included in the form of DC power flows in [9] where a social welfare maximization problem is solved, with the objective of the total user utility minus the total generation cost for day-ahead system scheduling. To handle the problem with large-scale integration of DR from small loads, a decomposition algorithm is developed based on the dual decomposition.

Another issue in those literature is that the multi-dimensional flexibility (MDF) that can be provided by different types of DR participants has not been captured. The MDF can include the following aspects of a flexible load: i) the total energy consumption requirement which can be within a certain range; ii) the power rating for consuming the energy which can be adjusted according to different conditions; iii) the time interval of the DR service which may also be flexible such as the charging of electric vehicles (EVs); and iv) the location of the flexible loads in the network which is also a very important aspect and the mobility of EVs has the potential of this locational flexibility. Some works in the literature have taken different dimensions of those flexibility into account [10]–[13] and various markets are designed for differentiated-energy services. In [10], a forward market framework for the deadline differentiated-energy services [13] is studied, where consumers consent to deferred supply of energy in exchange for a reduced per-unit energy price. In addition, a mechanism consisting of an earliest-deadline-first policy is proposed to solve the constrained stochastic optimal control problem. Moreover, they have shown that their method can achieve an efficient competitive equilibrium that simultaneously maximizes the supplier's profit and the social welfare. Similarly, some works have considered the market for other kinds of flexible energy services such as: i) rate-constrained energy services where the maximum rate at which the energy may be delivered to the consumers is considered as a key characteristic [12]; and ii) duration-differentiated energy services in which the flexibility resides in the fact that the power may be delivered at any time so long as the total duration of service is equal to the load's specified duration.

Different from those works in the literature where specific forward markets are designed for different flexible energy services, the present work proposes a unified market framework for MDF services. Specifically, each MDF service is composed of the following key attributes: i) a user specified service period (e.g., 10 a.m. to 4 p.m.); ii) the maximal load adjustment with respect to the base load reference (normal consumption) during the service period and iii) the maximal load adjustment per unit time slot (e.g., an hour). Moreover, since the transmission network constraints are taken into account, the locational information of the service is also captured. Then, we will introduce several key features of this framework.

1) *Flexibility Aggregator and Virtual Battery Model:* Since our flexibility market is considered at the transmission level, we assume that at each load bus, there exists an aggregator which represents the aggregate flexibility at this bus by a virtual battery model [14], [15]. Basically, the flexible load at each bus can be modeled by a battery which is characterized by an energy capacity and the charge/discharge rate limits. The detailed DR aggregation method is beyond the scope of this paper.
2) *Parametric DR bidding:* For each aggregator, apart from submitting the key parameters of its battery model, it should also submit a *parametric reward function* [16], [17] which is used for characterizing the remuneration requested by the aggregator. The form of this parametric reward function is defined by the market operator and the aggregator only needs to choose several parameters. Different from the simple bid consisting of a price-quantity pair in a traditional market, in our framework, the reward is a function of the actual flexibility accepted by the market operator.
3) *Market Clearing and Partial Acceptance:* The market clearing process is integrated into the traditional economic dispatch problem where the objective is to minimize the summation of the total generation cost plus the total payment of the MDF services for the aggregators. One important feature of our clearing process is that the flexibility bids can be *partially* accepted. This means that only a portion of flexibility service can be accepted. In contrast to a traditional DR market where a bid can either be totally accepted or rejected, our market design will definitely bring more flexibility to both the operator and aggregators. Moreover, the actual payment is dependent on the actual service that is accepted. Therefore, the evaluation for the complex MDF services is solved by this bidding-clearing process.

Many case studies are carried out to see how different bidding parameters will influence the results of the market clearing process. We find that for the same characteristics of the virtual batteries and the same bidding parameters in the reward function, the clearing results for the MDF bids at different buses can be quite different. This will in turn give some insights for the aggregators when they make their own bidding strategies.

The rest of this paper is organized as follows. The parametric bidding model and the formulation of the market clearing problem is introduced in Section II. Case studies are given in Section III followed by a conclusion in Section IV.

II. Parametric Bidding and Market Clearing

A. Parametric Bidding Model

Suppose we consider a transmission network with N buses and denote the set of all the buses as $\mathcal{N} = \{1, 2, \cdots, N\}$. Denote the set of those buses that will offer the MDF service as $\mathcal{F} \subseteq \mathcal{N}$ and the total time horizon is $\mathcal{T} = \{1, 2, \cdots, T\}$. For some aggregator $j \in \mathcal{F}$, the MDF bid consists of two parts. The first part is related to the virtual battery model and the main attributes are denoted as:

$$\boldsymbol{\psi}_j = \left(t_j^1, t_j^2, P_j^{min}, P_j^{max}, E_j^{min}, E_j^{max}, \{D_j(t)\}_{t\in[t_j^1, t_j^2]}\right).$$

In this bid, $t_j^1 \in \mathcal{T}$ and $t_j^2 \in \mathcal{T}$ define the user specified service period $[t_j^1, t_j^2]$ where $t_j^1 \leq t_j^2$. The aggregator also needs to submit its base load during the service period as a reference, which is denoted as $\{D_j(t)\}_{t\in[t_j^1, t_j^2]}$. Furthermore, $P_j^{max} \leq 0$ and $P_j^{min} \geq 0$ denote the maximal charging and discharging rate of this battery, respectively. Additionally, E_j^{min} and E_j^{max} denote the maximal net energy that can be stored into or released from this battery, respectively. We should note that here, both the storing and the releasing of energy is respective to the base load. Therefore, suppose at time t, the base load is

$D_j(t)$. Then, if we are *charging* an amount of $\Delta D > 0$ energy at this time slot, the actual load consumption is $D_j(t) - \Delta D$, which means the load is decreased. In the similar way, if we are *discharging* an amount of $\Delta D > 0$ energy, the actual load is $D_j(t) + \Delta D$, which means the load is increased. Evidently, if some energy storage devices are equipped at bus j, this kind of operation can be easily realized. However, some other loads can be modeled by this kind of virtual battery behavior such as thermostatically controlled loads (TCLs) [15]. As we have mentioned before, the market operator does not have to choose between accepting this bid or rejecting it. In fact, it can choose to accept only a portion of this bid. To be more specific, for each $\boldsymbol{\psi}_j$, four decision variables are associated with it with respect to the operator and they are denoted as: $\boldsymbol{\alpha}_j = (\alpha_j^{P1}, \alpha_j^{P2}, \alpha_j^{E1}, \alpha_j^{E2})$, which satisfy the following constraints:

$$P_j^{min} \le \alpha_j^{P1} \le 0 \le \alpha_j^{P2} \le P_j^{max}, \quad (1)$$

$$E_j^{min} \le \alpha_j^{E1} \le 0 \le \alpha_j^{E2} \le E_j^{max}. \quad (2)$$

Besides the first part, the aggregator also needs to submit a parametric reward function such that the actual payment $r_j(\boldsymbol{\alpha}_j)$ for the MDF service is a function of $\boldsymbol{\alpha}_j$. A simple example of the parametric reward function can be in linear form:

$$r_j(\boldsymbol{\alpha}_j) = \gamma_j^P(\alpha_j^{P2} - \alpha_j^{P1}) + \gamma_j^E(\alpha_j^{E2} - \alpha_j^{E1}).$$

In this example, each aggregator only needs to submit two parameters to the operator to evaluate the payment: γ_j^P and γ_j^E, which are the linear coefficients for the power rating limit and energy adjustment limit, respectively. We should emphasize that the form of this parametric reward function is defined by the operator for ease of clearing the MDF market and it is not limited to a linear function. For example, it can be a convex function of $\boldsymbol{\alpha}_j$.

B. Market Clearing

Denote the set of the transmission lines as $\mathcal{L}$ and the set of the generator buses as $\mathcal{G}$. Suppose that at each time slot t, the predicted load at bus $i \in \mathcal{D}$ is $D_i(t)$, where $\mathcal{D}$ is the set of load buses. Note that when $i \in \mathcal{F}$ and $t \in [t_i^1, t_i^2]$, there is no need to predict $D_i(t)$ since they are included in the bid submitted by aggregator i. For the generator at bus $i \in \mathcal{G}$ at time t, we assume that the generation cost is a quadratic function of the generation output $P_{G_i}(t)$:

$$C_i(P_{G_i}(t)) = a_i P_{G_i}(t)^2 + b_i P_{G_i}(t) + c_i.$$

Suppose there are G number of generators, F number of MDF aggregators and D number of load buses, then we can define a generator connection matrix $H_g \in R^{N\times G}$ such that its $(i,j)^{th}$ element is 1 if and only if generator j is located at bus i. Similarly, a load connection matrix $H_d \in R^{N\times D}$ and an aggregator connection matrix $H_f \in R^{N\times F}$ are defined such that their $(i,j)^{th}$ elements are 1 if and only if load j or aggregator j is located at bus i, respectively. Moreover, we denote the charging/discharging variable of aggregator i at time t as $P_{f_i}(t)$ and the corresponding vector as $\boldsymbol{P}_f(t) = (P_{f_1}(t), \cdots, P_{f_F}(t))^T$, where the superscript T is the transpose operator. Furthermore, the vector of $P_{G_i}(t)$ and $D_i(t)$ are denoted as $\boldsymbol{P}_G(t) = (P_{G_1}(t), \cdots, P_{G_G}(t))^T$ and $\boldsymbol{D}(t) = (D_1(t), \cdots, D_D(t)^T)$, respectively.

The day-ahead MDF market is cleared by solving the following optimization problem:

$$\begin{aligned}
\underset{\boldsymbol{P_G}, \boldsymbol{P}_f, \boldsymbol{\alpha}}{\text{minimize}} \quad & \sum_t \sum_{i\in\mathcal{G}} C_i(P_{G_i}(t)) + \sum_{i\in\mathcal{F}} r_i(\boldsymbol{\alpha}_i) \\
\text{subject to} \quad & \sum_{i\in\mathcal{G}} P_{G_i}(t) = \sum_{i\in\mathcal{D}} D_i(t) - \sum_{i\in\mathcal{F}} P_{f_i}(t), && \text{(3a)} \\
& \boldsymbol{P}_G^{min} \le \boldsymbol{P}_G(t) \le \boldsymbol{P}_G^{max}, && \text{(3b)} \\
& |\boldsymbol{\Gamma}(H_g \boldsymbol{P}_G(t) - H_d \boldsymbol{D}(t) + H_f \boldsymbol{P}_f(t))| \le \boldsymbol{f}^m, && \text{(3c)} \\
& E_i(0) = 0, && \text{(3d)} \\
& E_i(t+1) = E_i(t) + P_{f_i}(t), i\in\mathcal{F}, t=t_i^1, \ldots, t_i^2, && \text{(3e)} \\
& \alpha_i^{P1} \le P_{f_i}(t) \le \alpha_i^{P2}, i\in\mathcal{F}, t=t_i^1, \ldots, t_i^2, && \text{(3f)} \\
& \alpha_i^{E1} \le E_i(t) \le \alpha_i^{E2}, i\in\mathcal{F}, t=t_i^1, \ldots, t_i^2, && \text{(3g)} \\
& P_i^{min} \le \alpha_i^{P1} \le 0 \le \alpha_i^{P2} \le P_i^{max}, && \text{(3h)} \\
& E_i^{min} \le \alpha_i^{E1} \le 0 \le \alpha_i^{E2} \le E_i^{max}. && \text{(3i)}
\end{aligned}$$

In the objective, the first term is the total generation cost and the second term is the total MDF payment for all the aggregators. Constraint (3a) is the power balance constraint and (3b) requires that the output of each generator is within a range, where $\boldsymbol{P}_G^{min}$ and $\boldsymbol{P}_G^{max}$ denote the vectors of the lower and upper generation limits, respectively. In (3c), the $L \times N$ matrix $\boldsymbol{\Gamma}$ denotes the matrix of the generation shift factors where the $(l,i)^{th}$ element represents the change in the real power flow in branch l given a unit increase in the power injected at bus i [18]. Hence, the network line flow constraints are accounted for in (3c), where $\boldsymbol{f}^m$ denotes the vector of line limits. Constrains (3d)–(3i) are for the virtual batteries of the aggregators. To be specific, since the charging/discharging of batteries are with respect to the base load, the initial energy in the battery is zero as in (3d) and it can be increased or decreased as in (3e) depending on whether the charging or discharging decision is made. Constraints (3f) and (3g) are the power and energy limits for the batteries, respectively, and the limits $\boldsymbol{\alpha}_i$ can be determined after the market is cleared. In addition, (3h) and (3i) allow the market operator to have a partial acceptance of the MDF bids from all the aggregators. When $\alpha_i^{P1} = \alpha_i^{P2} = 0$ and $\alpha_i^{E1} = \alpha_i^{E2} = 0$ for some $i \in \mathcal{F}$, it means that the bid of aggregator i is rejected. After Problem (3) is solved, the market operator can determine how much a MDF bid will be accepted. The optimal dispatch of generators as well as the accepted virtual batteries can also be determined. Problem (3) is convex and hence, it can be solved efficiently by commercial solvers such as CVX [19].

III. Case Studies

In this section, we use a 6-bus transmission network [20] as an example to implement our day-ahead MDF market. Moreover, various case studies are carried out in order to find

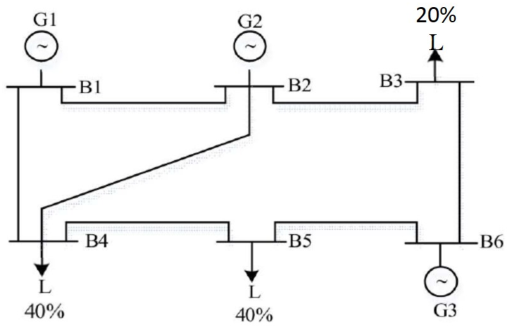


Fig. 1. 6-bus transmission network.

TABLE I
GENERATOR DATA

Index	P_G^{min}(MW)	P_G^{max}(MW)	a($/MW^{2}h)	b($/MWh)	c($)
G1	40	220	0.03	7	100
G2	10	200	0.07	10	104
G3	0	25	0.05	8	110

TABLE II
NETWORK INFORMATION

From Bus	To Bus	X (p.u.)	Flow Limit (MW)
1	2	0.170	60
1	4	0.258	70
2	3	0.037	190
2	4	0.197	200
3	6	0.018	180
4	5	0.037	190
5	6	0.140	180

TABLE III
TOTAL HOURLY LOAD OVER 24-H HORIZON

Hour	D(MW)	Hour	D(MW)	Hour	D(MW)
1	175	9	185	17	256
2	169	10	202	18	247
3	165	11	228	19	246
4	155	12	236	20	237
5	155	13	242	21	237
6	165	14	244	22	233
7	173	15	249	23	210
8	174	16	256	24	210

how different MDF bids from the aggregators will influence the result of the market clearing process.

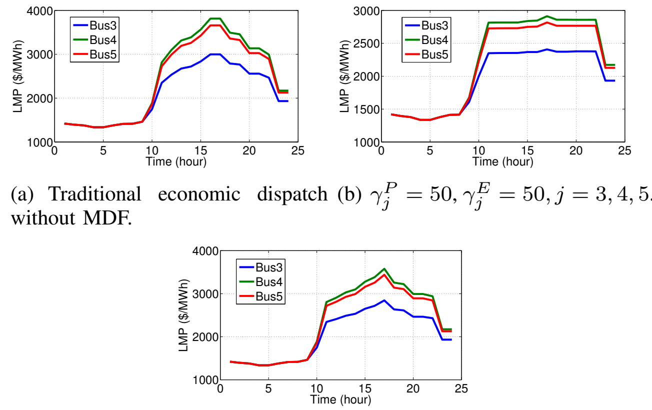
A. 6-bus Transmission Network

The topology of the 6-bus system is shown is Fig. 1. There are three generators located at Buses 1, 2 and 6, respectively. The other three buses are load buses and every load bus also has an aggregator to participate into the MDF market. The percentage indicated in the figure at each load bus represents the percentage of the power consumption at each load with respect to the system's total load. The characteristics of generators, network information, and the total hourly load over 24-h horizon are given in Tables I–III, respectively. The power base is 100MVA.

For the three aggregators at the load buses, the values of their bids (ψ_j) are given in p.u. in Table IV. We assume that the characteristics of the virtual batteries are the same for all the aggregators, except for their available service time. Next, we will use different bidding values of $\boldsymbol{\alpha}$ to see the clearing result of the MDF market.

TABLE IV
MDF BIDS IN (P.U.)

Index	t_j^1	t_j^2	P_j^{min}	P_j^{max}	E_j^{min}	E_j^{max}
B3	13	19	-0.1	0.3	-0.3	0.5
B4	9	16	-0.1	0.3	-0.3	0.5
B5	16	23	-0.1	0.3	-0.3	0.5



(a) Traditional economic dispatch without MDF. (b) $\gamma_j^P = 50, \gamma_j^E = 50, j = 3, 4, 5.$

(c) γ_j^P=500, γ_j^E=500, j=3, 4, 5.

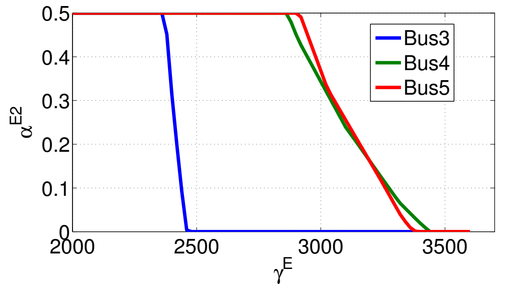
Fig. 2. LMPs for load buses in three cases.

TABLE V
MDF CLEARING RESULTS

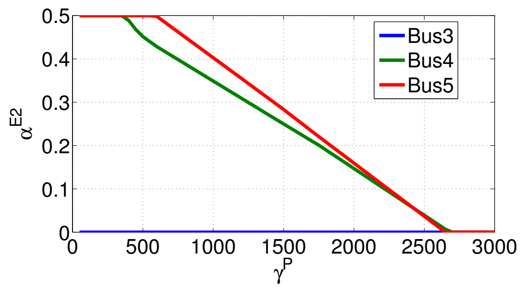
	$\gamma_j^P = 50, \gamma_j^E = 50$		$\gamma_j^P = 500, \gamma_j^E = 500$	
	α_j^{P2}	α_j^{E2}	α_j^{P2}	α_j^{E2}
B3	0.129	0.5	0.095	0.5
B4	0.179	0.5	0.134	0.5
B5	0.156	0.5	0.085	0.5

B. Market Clearing Result

In the first case, we solve the problem without any MDF aggregator. Then Problem (3) is reduced to a traditional economic dispatch problem. In the second case, the parameters for the bidding reward functions are as follows: $\gamma_3^P = \gamma_4^P = \gamma_5^P = 50$ and $\gamma_3^E = \gamma_4^E = \gamma_5^E = 50$. In the third case, we increase both γ_j^P and γ_j^E by 10 times which means they are all equal to 500. For these three cases, we compare the hourly locational marginal prices (LMPs) at Buses 3, 4, and 5 in Fig. 2. Also, the cleared values of $\boldsymbol{\alpha}$ are shown in Table V. Note that the values of α_j^{P1} and α_j^{E1} are not shown. This is because they are always equal to zero in our case studies. The reason for this is because if they are not equal to zero, it means that the system needs the virtual battery to discharge some energy, which is equivalent to increasing the load. However, in our current setting, increasing the load will not bring benefit to the system. Nevertheless, these values may be nonzero when the random generation outputs of RES (e.g., solar panels and



(a) Cleared α^{E2} for different γ^E, with $\gamma^P = 500$.



(b) Cleared α^{E2} for different γ^P, with $\gamma^E = 2900$.

Fig. 3. Cleared α^{E2} for different γ^P and γ^E.

wind turbines) are included. In that case, if the RES outputs are much larger than the base load and the traditional generators have ramping limits, then increasing the load by discharging the virtual batteries will be beneficial to the system.

In Fig. 2(a), we observe that without MDF, the LMPs for the load buses vary a lot between 10:00 to 23:00. This is due to the large variation of loads and the peak-to-average ratio in load demand is very high. In Fig. 2(b), Fig.2(c) and Table V, it can be seen that in both cases, the accepted values of α_j^{E2} for all the three buses always reach their maximum values (i.e., 0.5). This is reasonable because the payment for the load reduction is much lower than the generation cost, so to accept all of them will minimize the system cost. However, the accepted power limits α_j^{P2} decrease by 26.4%, 25.2% and 45.5% for Buses 3, 4 and 5, respectively when the bidding parameters in the reward function increase from 50 to 500. If we compare the change of the LMPs in Fig. 2(b) and Fig. 2(c), it can be noted that, for the same α_j^{E2}, on the one hand, larger α_j^{P2} can flatten the LMPs very well, which is good since the variations as well as the average values of LMPs are decreased. On the other hand, smaller α_j^{P2} can also reduce LMPs, but the effect of flattening LMPs is much less than the previous case.

After analyzing how different bidding parameters in the reward function can affect LMPs, next, we will show what would be the difference of cleared values of α_j^{E2} for the MDF aggregators when they all have the same bidding parameters. To be specific, firstly, we fix $\gamma_3^P = \gamma_4^P = \gamma_5^P = 500$ and change the value of γ^E from 2000 to 3600, where $\gamma^E = \gamma_3^E = \gamma_4^E = \gamma_5^E$. The results for the cleared values of $\alpha_j^{E2}, j = 3, 4, 5$ are shown in Fig. 3(a). It can be seen that although the characteristics of the three virtual batteries and the parameters in the reward functions are identical for all the three buses, their cleared values of α_j^{E2} are quite different when γ^E is increased. In detail, they are all equal to the maximum value (i.e., 0.5) when γ^E is smaller than 2360. As γ^E is further increased, the cleared values of α_j^{E2} for the three buses decrease in a sequential manner. Bus 3 is the first one whose α_j^{E2} starts to decrease and it reaches zero even when the other two buses still have the maximal α_j^{E2}. This shows that the MDF bid of Bus 3 is the least valuable one among all the bids for the market operator. In contrast, the cleared values of α_j^{E2} of Buses 4 and 5 start to decrease when γ^E is equal to 2860 and 2900, respectively. This means these two buses have higher "bargaining power" in the market since with higher bidding reward functions for the MDF services, they can still be accepted by the market. Therefore, the results of the market clearing process will reflect the true interest and values of different MDF bids from the perspective of the market operator. These different values of the MDF bids at different buses can be related to many factors such as the loads, locations and serving periods, etc.

Secondly, we fix $\gamma_3^E = \gamma_4^E = \gamma_5^E = 2900$ and change the value of γ^P from 50 to 3000, where $\gamma^P = \gamma_3^P = \gamma_4^P = \gamma_5^P$. The results for the cleared values of $\alpha_j^{E2}, j = 3, 4, 5$ are shown in Fig. 3(b). It is noted that α_j^{E2} decreases as γ^P increases. This means that α_j^{E2} is not only influenced by the bidding parameters for the battery energy limits γ_j^E, but it is also affected by the bidding parameters for the power limits γ_j^P. As a result, when MDF aggregators submit bids to the market, they should jointly make decisions on γ_j^P and γ_j^E, because if any one of the two bidding parameters are not chosen in a right way, it may result in that only a little portion of their bids can be accepted.

IV. Conclusion

We have studied a day-ahead market for the MDF services in the transmission network. Each MDF aggregator needs to submit the characteristics of its virtual battery which represent the aggregate load flexibility. Also, a parametric reward function which represents the expected payment should be submitted at the same time. The market clearing process is modeled by a convex optimization problem and a key feature of this MDF market is that the partial acceptance of a bid is allowed, which means that the system operator can accept a portion of the bid instead of the whole. We find that this MDF market can help flatten the LMPs across the peak load period, and with the same energy adjustment capacity of the virtual batteries, different power limits within a single time slot can lead to different flattening effect to LMPs. Moreover, we show that even if the MDF bids at different buses are the same, the clearing results can be quite different, which reflect the practical values of different load flexibilities from the perspective of the system operator. Therefore, this market can finally help the operator to have the optimal strategy of accepting different MDF bids such that the system cost can be minimized. As for the future work, we will further involve the RES and the uncertainty of the load into our problem to see how much benefit this MDF market can bring to the system under the more complicated scenarios.

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